## Part 3: Limits to $\alpha_m$ and $n_e$

The limiting factor to  $n_e$ , and, consequently  $\alpha_m$  is the probability of survival of an individual with the maximum fitness value in the population ( $P_s(m_{max})$ ).

Since no probability of survival can be above 1.0 (100%), we have:

$$F(m_{max}) = \alpha_m P_s(m_{max})$$
$$\frac{F(m_{max})}{\alpha_m} = P_s(m_{max}) \le 1$$
$$\frac{F(m_{max})}{\alpha_m} \le 1$$
$$\alpha_m \ge F(m_{max})$$

And hence,

$$F(m_{max}) \le \alpha_m = \frac{\sum F(M)}{n_e}$$
$$F(m_{max}) \le \frac{\sum F(M)}{n_e}$$
$$\boxed{n_e \le \frac{\sum F(M)}{F(m_{max})}}$$

or,

$$n_{e} \leq n_{m} \frac{F(m_{avg})}{F(m_{max})}$$

A fitness scalar must therefore be at least as large as the maximum fitness value that is present in the population. Likewise, the number of expected survivors cannot exceed the total fitness divided by the maximum fitness value included in its population.