Part 5: Natural Selection

The total final expected fitness of a population of genomes after selection will depend on the fitness values of the genomes that make up the population. In general, the expected fitness contribution per individual will be that individual's fitness times its probability of survival. The total final expected fitness of a population of genomes after selection will thus be:

$$\sum F_e(M')=\sum (F(m_x)P_s(m_x))$$

$$\sum F_e(M')=\frac{\sum (F(m_x)^2)}{\alpha_m}$$

$$\sum F_e(M')=\frac{\sum (F(m_x)^2)}{\sum F(M)}n_e$$

$$F_e(m'_{avg})=\frac{\sum (F(m_x)^2)}{\sum F(M)}$$

$$\boxed{F_e(m'_{avg})=\frac{\sum (F(M)^2)}{\sum F(M)}}$$

Given that the standard variance of the population M is:

$$\sigma_m^2=\frac{1}{n_m}(\sum_{i=1}^{n_m}(F(m_i)-F(m_{avg}))^2)$$

$$\sigma_m^2=\frac{1}{n_m}(\sum_{i=1}^{n_m}(F(m_i)^2+F(m_{avg})(F(m_{avg})-2F(m_i))))$$

$$\sigma_m^2=\frac{1}{n_m}(\sum (F(M)^2)+F(m_{avg})\sum_{i=1}^{n_m}(F(m_{avg})-2F(m_i)))$$

$$\sigma_m^2=\frac{1}{n_m}(\sum (F(M)^2)+F(m_{avg})(F(m_{avg})n_m-2\sum_{i=1}^{n_m}F(m_i)))$$

$$\sigma_m^2=\frac{1}{n_m}(\sum (F(M)^2)+F(m_{avg})(\sum F(M)-2\sum F(M)))$$

$$\sigma_m^2=\frac{1}{n_m}(\sum (F(M)^2)-F(m_{avg})\sum F(M))$$

$$\sigma_m^2 = \frac{1}{n_m}\left(\sum (F(M)^2) - n_m F(m_{avg})^2\right)$$

$$\boxed{\sigma_m^2 = \frac{\sum (F(M)^2)}{n_m} - F(m_{avg})^2}$$

Which gives us:

$$\sigma_m^2 + F(m_{avg})^2 = \frac{\sum (F(M)^2)}{n_m}$$

$$\boxed{\sum (F(M)^2) = n_m(\sigma_m^2 + F(m_{avg})^2)}$$

Substituting in our original equation, we get:

$$F_e(m'_{avg}) = \frac{n_m(\sigma_m^2 + F(m_{avg})^2)}{\sum F(M)}$$

$$F_e(m'_{avg}) = \frac{n_m(\sigma_m^2 + F(m_{avg})^2)}{n_m F(m_{avg})}$$

$$\boxed{F_e(m'_{avg}) = F(m_{avg}) + \frac{\sigma_m^2}{F(m_{avg})}}$$

Thus, the expected average fitness after selection depends on the average fitness before selection and the standard variance from that average (the standard variance before selection).

Selection according to fitness, then, (i.e. our model of Natural Selection,) will raise the average expected fitness by $\sigma_m^2 / F(m_{avg})$, the standard variance over the average fitness of the group that the selection is acting upon. Since the standard variance is always positive, we must have:

$$\boxed{F_e(m'_{avg}) \geq F(m_{avg})}$$

However, selection can be of no advantage where there is no variation in the population. If $\sigma_m^2 = 0$ there will be no expected change in the average fitness of the population after selection takes place.